Final Exam

1. Simplify: a. $\left(3x^4-7x^3+7x^2+8x-4\right)+\left(8x^3-11x^2+3x+12\right)$

$$\begin{array}{l} 3x^4-7x^3+7x^2+8x-4 \\ \underline{0x^4+8x^3-11x^2+3x+12} \\ \boxed{3x^4+x^3-4x^2+11x+8} \end{array}$$

b. $\left(6x^4-5x^2+7x-12\right)-\left(9x^4-5x^3+3x^2+6x+15\right)$

$$\begin{array}{l} 6x^4+0x^3-5x^2+7x-12 \\ \underline{-(9x^4-5x^3+3x^2+6x+15)} \\ \boxed{-3x^4+5x^3-8x^2+x-27} \end{array}$$

2. Multiply: a. $6+\left(8x-7\right)\left(5x^2+7x\right)$

$$6+\left(40x^3+56x^2-35x^2-49x\right)\rightarrow 6+\left(40x^3+21x^2-49x\right)\rightarrow \boxed{40x^3+21x^2-49x+6}$$

b. $(8-3x)(7x-8)$

$$56x-64-21x^2+24x\rightarrow \boxed{-21x^2+80x-64}$$

3. Multiply: a. $(2x^2-2x)^2$

$$(2x^2-2x)(2x^2-2x)\rightarrow(4x^4-4x^3-4x^3+4x^2)\rightarrow\boxed{4x^4-8x^3+4x^2}$$

b. $(A-B)^3$

$$(A-B)\bullet(A-B)\bullet(A-B)\rightarrow(A^2-2AB+B^2)(A-B)\rightarrow(A^3-A^2B-2A^2B+2AB^2+AB^2-B^3)\rightarrow$$
$$\boxed{A^3-3A^2B+3AB^2-B^3}$$

4. Factor: a. $18x^7+12x^5-3x^3$

$$\boxed{3x^3(6x^4+4x^2-1)}$$

b. x^2-49

$$\boxed{(x-7)(x+7)}$$

5. Factor: a. $x^2 - 3x - 54$

$(x^2 - 9x + 6x - 54) \to x(x-9) + 6(x-9) \to \boxed{(x-9)(x+6)}$

b. $6x^2 - 11x - 10$

$(6x^2 + 4x - 15x - 10) \to 2x(3x+2) - 5(3x+2) \to \boxed{(3x+2)(2x-5)}$

6. Factor: a. $x^3 - 125$

$(x^3 - 5^3) \to \boxed{(x-5)(x^2 + 5x + 25)}$

b. $x^3 + 64$

$(x^3 - 4^3) \to \boxed{(\mathrm{x} - 4)(\mathrm{x}^2 + 4\mathrm{x} + 16)}$

7. Solve: a. $x^2 - 3x - 40 = 0$

$(x^2 - 8x + 5x - 40) \to x(x-8) + 5(x-8) \to (x-8)(x+5) = 0 \to \boxed{x = -5{,}8}$

b. $4x^2 + x - 10 = (x-2)(x+1)$

$4x^2 + x - 10 = (x^2 + 1x - 2x - 2) \to 4x^2 + x - 10 = x^2 - x - 2 \to 4x^2 + x - 10 - x^2 + x + 2 = 0 \to$
$3x^2 + 2x - 8 = 0 \to 3x(x+2) - 4(x+2) = 0 \to (3x-4)(x+2) = 0 \to \boxed{x = -2, \frac{4}{3}}$

8. Find the equation of the line containing the points (-3, -11) and (-5, 9)

$m = \frac{y-y}{x-x}$ Is the formula to find the slope. $\frac{-11-(9)}{-3-(-5)} = \frac{-20}{2} = m = -10$
Now solve for b in the line formula y = mx + b

$-11 = -10(-3) + b \to -11 = 30 + b \to b = -41$
Now plug both b and m into the formula y = mx + b

$\boxed{y = -10x - 41}$

9. Find the midpoint of the line segment connecting the pair of points (-3,-11) and (-5, 9).

The midpoint formula for a straight line is $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \rightarrow Midpoint = (x, y)$

$$\frac{-3+(-5)}{2}, \frac{-11+9}{2} \rightarrow \frac{-8}{2}, \frac{-2}{2} \rightarrow \boxed{Midpoint = (-4,-1)}$$

10. Find the distance between the points (-3, -11) and (-5, 9).

The distance formula is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$d = \sqrt{(-3-(-5)^2 + (-11-9)^2} \rightarrow d = \sqrt{(2)^2 + (-20)^2} \rightarrow d = \sqrt{4+400} \rightarrow d = \sqrt{404} \rightarrow d = \sqrt{4 \bullet 101} \rightarrow$$

$$\boxed{d = 2\sqrt{101}}$$

11. Multipy: $\frac{x+6}{x-3} \bullet \frac{x-6}{x+3}$

$$\frac{x^2 - 6x + 6x - 36}{x^2 + 3x - 3x - 9} \rightarrow \boxed{\frac{x^2 - 36}{x^2 - 9}}$$

12. Multiply: $\frac{x^2 - 6x - 16}{x^2 - 64} \bullet \frac{x^2 + x - 2}{x^2 + 4x + 4}$

$$\frac{x^2 - 8x + 2x - 16}{(x-8)(x+8)} \bullet \frac{x^2 + 2x - 1x - 2}{(x+2)(x+2)} \rightarrow \frac{x(x-8) + 2(x-8)}{(x-8)(x+8)} \bullet \frac{x(x+2) - 1(x+2)}{(x+2)(x+2)} \rightarrow \frac{(x+2)(x-8)}{(x-8)(x+8)} \bullet \frac{(x+2)(x-1)}{(x+2)(x+2)} \rightarrow$$

$$\boxed{\frac{(x-1)}{(x+8)}}$$

13. Add: $\frac{x+9}{x-3} + \frac{x-12}{x+3}$

$$\frac{x+9}{x-3} \bullet \frac{x+3}{x+3} + \frac{x-12}{x+3} \bullet \frac{x-3}{x-3} \rightarrow \frac{x^2 + 9x + 3x + 27}{x^2 - 3x + 3x - 9} + \frac{x^2 - 12x - 3x + 36}{x^2 - 3x + 3x - 9} \rightarrow \frac{x^2 + 12x + 27}{x^2 - 9} + \frac{x^2 - 15x + 36}{x^2 - 9} \rightarrow$$

$$\boxed{\frac{2x^2 - 3x + 63}{x^2 - 9}}$$

14. Subtract: $\dfrac{x-3}{x^2-5x+4}-\dfrac{x+7}{(x^2-1)}$

$$\frac{x-3}{x^2-4x-1x+4}-\frac{x+7}{(x-1)(x+1)}\rightarrow\frac{x-3}{(x-4)(x-1)}-\frac{x+7}{(x-1)(x+1)}\rightarrow\frac{(x-3)(x+1)}{(x-4)(x-1)(x+1)}-\frac{(x+7)(x-1)}{(x-4)(x-1)(x+1)}\rightarrow$$

$$\frac{x^2-2x-3}{x^3-4x^2-x+4}-\frac{x^2+3x-28}{x^3-4x^2-x+4}\rightarrow\boxed{\frac{-5x+25}{x^3-4x^2-x+4}}$$

15. Simplify: $\dfrac{\dfrac{b+5}{b+2}-\dfrac{5}{b+3}}{\dfrac{3}{b+2}+\dfrac{b+4}{b+3}}$

$$\frac{\dfrac{(b+5)(b+3)}{(b+2)(b+3)}-\dfrac{5(b+2)}{(b+2)(b+3)}}{\dfrac{3(b+3)}{(b+2)(b+3)}+\dfrac{(b+4)(b+2)}{(b+2)(b+3)}}\rightarrow\frac{\dfrac{b^2+8b+15}{b^2+5b+6}-\dfrac{(5b+10)}{b^2+5b+6}}{\dfrac{3b+9}{b^2+5b+6}+\dfrac{b^2+6b+8}{b^2+5b+6}}\rightarrow\frac{\dfrac{b^2+3b+5}{b^2+5b+6}}{\dfrac{b^2+9b+17}{b^2+5b+6}}\rightarrow\frac{b^2+3b+5}{b^2+5b+6}\bullet\frac{b^2+5b+6}{b^2+9b+17}\rightarrow$$

$$\boxed{\frac{b^2+3b+5}{b^2 9b+17}}$$

16. Solve: $\dfrac{2x}{x+2}+3x=\dfrac{-5}{x-3}$

$$\frac{2x(x-3)}{(x+2)(x-3)}+\frac{3x(x+2)(x-3)}{(x+2)(x-3)}=\frac{-5(x+2)}{(x+2)(x-3)}\rightarrow\frac{3x^3-x^2-24x}{x^2-x-6}=\frac{-5x-10}{x^2-x-6}\rightarrow\frac{3x^3-x^2-24x}{x^2-x-6}+\frac{5x+10}{x^2-x-6}=0\rightarrow$$

$3x^3-x^2-19x+10=0\rightarrow$ $\boxed{\text{Can not solve, no solution}}$

17. Solve: $\dfrac{9}{x^2+4x-45}=\dfrac{7}{x+9}-\dfrac{7}{x-5}$

$$\frac{9}{(x+9)(x-5)}=\frac{7(x-5)}{(x+9)(x-5)}-\frac{7(x+9)}{(x+9)(x-5)}\rightarrow\frac{9}{(x+9)(x-5)}=\frac{7x-35}{(x+9)(x-5)}-\frac{7x+63}{(x+9)(x-5)}\rightarrow$$

$$\frac{9}{(x+9)(x-5)}\neq\frac{98}{(x+9)(x-5)}$$

$\boxed{9\neq 98\quad\text{no solution}}$

18. a. State the quadratic formula. $\boxed{\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

b. Solve the quadratic equation $3x^2 + 6x - 7 = 0$

$$\frac{-6 \pm \sqrt{6^2 - 4(3)(-7)}}{2(3)} \rightarrow \frac{-6 \pm \sqrt{36+84}}{6} \rightarrow \frac{-6 \pm \sqrt{120}}{6} \rightarrow \frac{-6 \pm \sqrt{4 \bullet 30}}{6} \rightarrow \frac{-6 \pm 2\sqrt{30}}{6} \rightarrow \frac{-6}{6} \pm \frac{2\sqrt{30}}{6} \rightarrow$$

$$\boxed{-1 \pm \frac{\sqrt{30}}{3}}$$

19. Solve: a. $\sqrt{3-5x} - 22 = -12$

$$\sqrt{3-5x} = 10 \rightarrow (\sqrt{3-5x})^2 = (10)^2 \rightarrow 3 - 5x = 100 \rightarrow -5x = 97 \rightarrow \boxed{x = -\frac{97}{5}}$$

b. $\sqrt{x+6} + \sqrt{x+2} = 2$

$$\sqrt{x+6} = -\sqrt{x+2} + 2 \rightarrow (\sqrt{x+6})^2 = (-\sqrt{x+2} + 2)(-\sqrt{x+2} + 2) \rightarrow x + 6 = x + 2 - 4\sqrt{x+2} + 4 \rightarrow -4\sqrt{x+2} = 0 \rightarrow$$
$$(-4\sqrt{x+2})^2 = (0)^2 \rightarrow 16(x+2) = 0 \rightarrow 16x + 32 = 0 \rightarrow 16x = -32 \rightarrow$$

$\boxed{x = -2}$

20. Simplify: $\sqrt[3]{8x^4 y^6 z^{12}}$

$$\sqrt[3]{2^3 x^4 y^6 z^{12}} \rightarrow \boxed{2xy^2 z^4 \sqrt[3]{x}}$$

21. Simplify: $(2\sqrt{3} + 4)(3\sqrt{3} - 1)$

$$6\sqrt{3^2} - \sqrt{3} + 12\sqrt{3} - 4 \rightarrow (6 \bullet 3) + 10\sqrt{3} - 4 \rightarrow 18 - 4 + 10\sqrt{3} \rightarrow \boxed{14 + 10\sqrt{3}}$$

22. Simplify: a. i^2

$\boxed{i^2 = -1}$

b. i^{25}

$\boxed{i^{25} = i}$

23. Simplify: $(2+i)+(2-i)(3+2i)$

$(2+i)+(6+4i-3i-2i^2) \rightarrow (2+i)+(6+i-2(-1)) \rightarrow (2+i)+(8+i) \rightarrow \boxed{10+2i}$

24. Simplify: a. $(2+i)^2+(2-i)(7+4i)$

$(2+i)(2+i)+(14+8i-7i-4i^2) \rightarrow (4+4i+i^2)+(14+i-4i^2) \rightarrow (4+4i-1)+(14+i-4(-1)) \rightarrow$
$3+4i+18+i \rightarrow$

$\boxed{21+5i}$

b. $\dfrac{5+2i}{3-2i}$

$$\frac{5+2i}{3-2i} \bullet \frac{3+2i}{3+2i} \rightarrow \frac{15+10i+6i+4i^2}{9-4i^2} \rightarrow \frac{15+16i+4(-1)}{9-4(-1)} \rightarrow \boxed{\frac{11+16i}{13}}$$

25. What is meant by the x- intercepts of a function? Find the x- intercepts of the quadratic function:
$f: f(x)=-2x^2-4x+30$

a. The x – intercepts of a function are the x coordinates on the x-axis that a horizontal line can intersect.

b. Find the x-intercepts of the quadratic function $f: f(x)=-2x^2-4x+30$

$-2x^2-10x+6x+30 \rightarrow -2x(x+5)+6(x+5) \rightarrow (-2x+6)(x+5)=0 \rightarrow (-2x+6)=0, (x+5)=0 \rightarrow$
$-2x=-6 \rightarrow x=\dfrac{-6}{-2} \rightarrow x=3, x=-5$

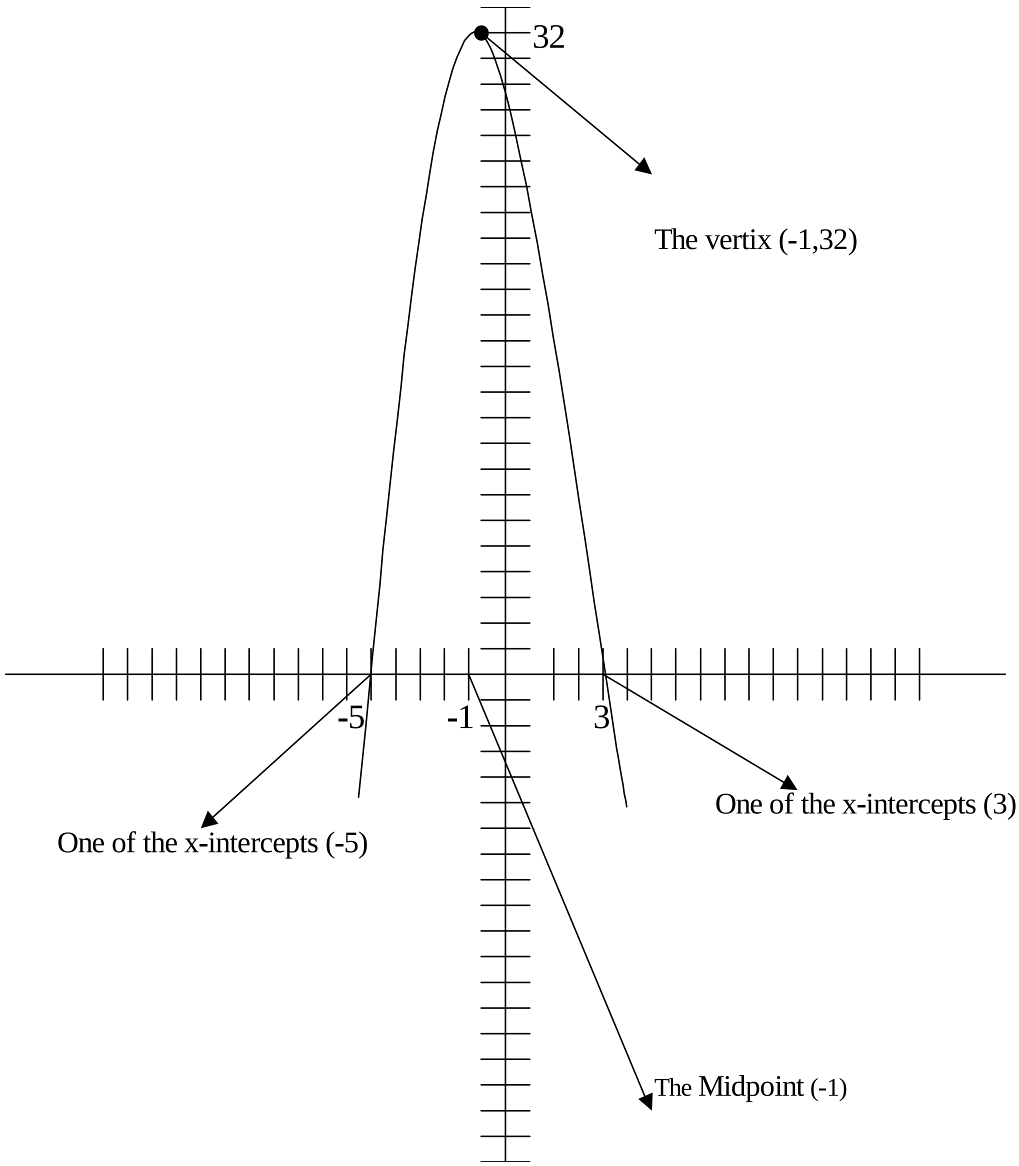
$\boxed{f(x)=3,-5}$

26. What is meant by the vertex of a function? Find the vertex of the function f, where ..
$f(x)=-2x^2-4x+30$

a. The vertex of a function is the mini or maxi of the parabola.

b. $\dfrac{-(-4)}{2(-2)} \rightarrow \dfrac{4}{-4} \rightarrow midpo\,\text{int}=-1 \rightarrow f(x)=-2(-1)^2-4(-1)+30 \rightarrow -2+4+30 \rightarrow \max=32\ \boxed{V=(-1,32)}$

27. Graph the function f, where $f(x) = -2x^2 - 4x + 30$



Formulas…….

1. Midpoint for a straight line: $\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

This formula is used to find the average of paired coordinates i.e..(-2,4) and (4, 8).
The answer should look like, the midpoint is (1, 6)

2. The distance formula is: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

This formula is used to find the distance between points, and all you have to do is plug in the x and y coordinates given and solve.

3. The slope formula is: $\frac{x_1 - x_2}{y_1 - y_2}$

This formula will give you the m value for the line formula y = mx +b. All you have to do is plug in the x and y coordinates given and solve the slope formula.

4. The formula for a line is: y = mx +b

When the question asks "Find the equation of the line containing points….", you first find the slope by using the slope formula and then plug the slope and one of the (x, y) coordinates given and solve for b. Once you have solved for b, rewrite the straight-line formula using the slope and b value found. See example….

Example: Find the equation of the line containing the points (1, 2) and (-5, 5)

The slope…. $\frac{1-(-5)}{2-(5)} \rightarrow \frac{6}{-3} \rightarrow -2 \rightarrow m = -2$

Now using the x and y coordinates (1,2) and the slope of –2 to solve for b…

$2 = -2(1) + b \rightarrow 2 = -2 + b \rightarrow b = 4$

Now rewrite the line formula using the values you found….

Your answer is … $\boxed{y = -2x + 4}$

5. The quadratic formula is … $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

6. The <u>midpoint</u> equation of <u>x-intercept</u> points is… $\frac{-b}{2a}$

This will be associated with a function i.e. … $f(x) = x^2 + 2x + 4$

The midpoint is found by plugging in the A and B values found in the function and plugging them into the midpoint formula.

7. The <u>vertex</u> of a function is found by plugging the midpoint answer as the x value into the function and solving the function. Once solved the midpoint and the y point (the final answer for the equation) are written as an ordered pair. The ordered pair is the answer for the vertex. See example…

Note: The vertex of a function is the same thing as the mini or max of the parabola

Example…

$f(x) = x^2 + 2x + 4$

The midpoint….. $\frac{-2}{2(1)} \rightarrow \frac{-2}{2} \rightarrow -1$, <u>Midpoint or x = -1</u>

Now that we know the midpoint is –1, so now solve the equation.. $f(x) = x^2 + 2x + 4$

$f(x) = (-1)^2 + 2(-1) + 4 \rightarrow 1 - 2 + 4 \rightarrow 3$, $f(x) = 3$ or it can be written as y = 3

The x value is –1 and the y value is 3, so the answer for the vertex is…v = (-1, 3)

8. $i = \sqrt{-1}$ All exponents of i can only equal 1, i, -1,or $-i$. Always start count with i.