Math 107 online practice test 3 solutions

Your exam will contain 16 questions selected from the following problem types.

1. From the following equations,

i)	$y(x) = -7x + 4$	ii)	$y(x) = -3(x - 3)^2 - 4$
iii)	$y(x) = 3(x - 3)(x + 4)$	iv)	$y(x) = 4x - 7$
v)	$y(x) = -4x + 7$	vi)	$y(x) = -5(x - 3)(x + 4)$

identify the equation of

a) a linear function which has slope 4 and y-intercept (0,-7) **iv**

b) a quadratic function which opens downwards and has a vertex at (3, -4) **ii**

2. From the following equations,

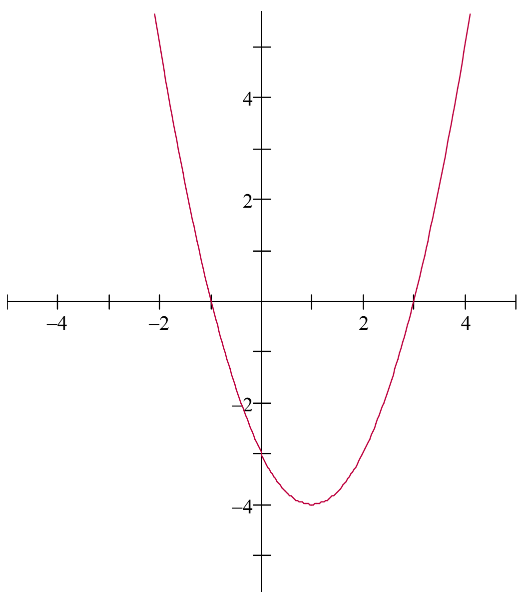
i)	$y(x) = x^4$	ii)	$y(x) = (x + 3)(x - 4)(x + 7)(3x - 5)$
iii)	$(x - 3)^2 + (y + 4)^2 = 64$	iv)	$y(x) = (x - 3)(x + 4)(x + 2)^2$
v)	$(x + 3)^2 + (y - 4)^2 = 8$	vi)	$(x + 3)^2 + (y - 4)^2 = 64$

identify the equation of

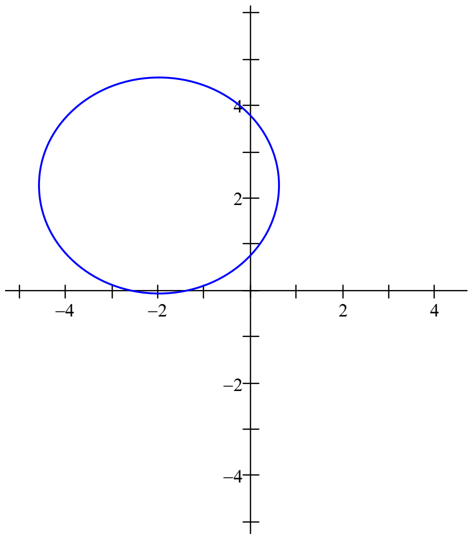
a) a circle which has radius 8 and center (-3, 4) **vi**

b) a function which has four x-intercepts. **ii**

3. Graph the function $y(x) = x^2 - 2x - 3$ for the domain $\{x \mid -2 \le x \le 4\}$



4. Graph the relation $(x + 2)^2 + (y - 3)^2 = 9$



5. Explain why the questions "Solve: $-3x^2 + 6x + 24 = 0$" and "Find the x-intercepts of the quadratic function f: $f(x) = -3x^2 + 6x + 24$" necessarily have the same values of x as solutions.

   The zeros of the equation *are* the x-intercepts.

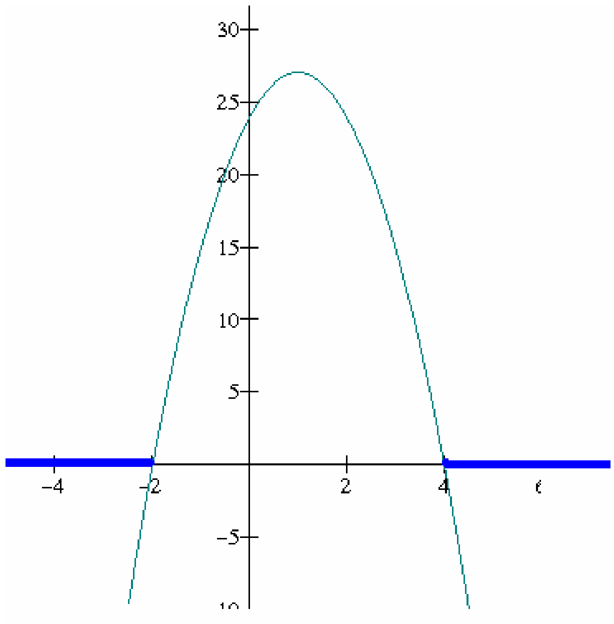
6. Find the x-intercepts of the quadratic function f: $f(x) = -3x^2 + 6x + 24$

   We solve $-3x^2 + 6x + 24 = 0$:
   $-3x^2 + 6x + 24 = -3(x^2 - 2x - 8) = -3(x - 4)(x + 2)$
   So we have zeros at x = -2, 4. Therefore, the x-intercepts are at (-2,0) and (4,0)

7. Find the vertex of the function f, where $f(x) = -3x^2 + 6x + 24$

   The vertex is given by (-b/2a, f(-b/2a)).
   Since b = 6 and a = -3, -b/2a = -6/2(-3) = 1
   f(1) = -3 + 6 + 24 = 29.
   The vertex of the quadratic function is at (1,29). Since a<0, the parabola opens downwards, so the vertex is a maximum.

8. Sketch the function f, where $f(x) = -3x^2 + 6x + 24$. On your sketch, shade the interval on x which corresponds to the solution set of the inequality $-3x^2 + 6x + 24 < 0$



9. State the domain and the range of the function f given in question 6.

The domain is all real numbers. The range is all real numbers less than or equal to 29.

For questions 10, 11, 12 use the following for functions f, g and h:

f(x) = 4 + 3x, g(x) = $2x^2 - 7$, h(x) = $\frac{1}{x^2}$

10. Find a) (fg)(3)

f(3) = 4 + 3(3) = 13 g(3) = 2(9) – 7 = 11.
Therefore (fg)(3) = 13(11) = 143.

b) (f – g)(x) = (4 + 3x) – ($2x^2 - 7$) = $-2x^2 + 3x + 11$

11. Solve: (f – g)(x) = 0

(f – g)(x) = 0 ⇒ $-2x^2 + 3x + 11 = 0$
This doesn't factor, so we use the quadratic formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4\cdot(-2)\cdot 11}}{2(-2)} = \frac{-3 \pm \sqrt{97}}{-4} = \frac{3 \pm \sqrt{97}}{4}$$

12. Find a) g ∘ h (x)

g ∘ h (x) = $g(1/x^2) = 2(1/x^2)^2 - 7 = 2/x^4 - 7$

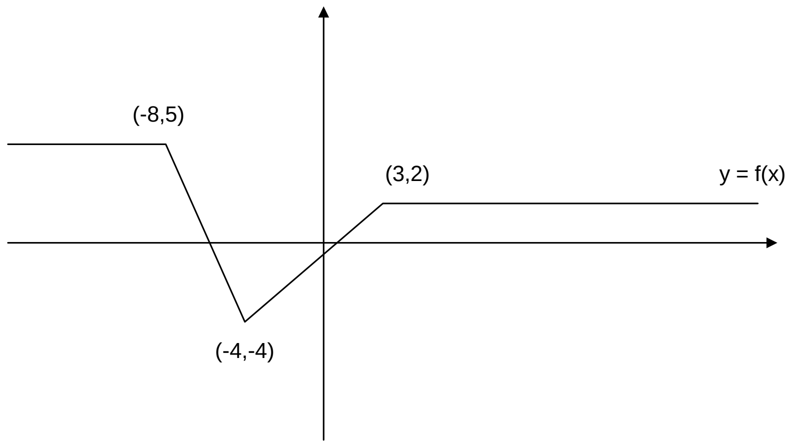
b) h ∘ h (x)

h ∘ h (x) = $h(1/x^2) = 1 / (1/x^2)^2 = 1/(1/x^4) = x^4$

13. The graph of the function y = f(x) is drawn below. Describe in words the effects on the graph of the following transformations.

a) y = -3f(x) vertical stretch of factor 3, reflection in x-axis

b) y = f(x + 3) horizontal translation left 3 places



14. Referring to the graph of y = f(x) in question 13, draw a graph of the transformed function y = -f(x – 2) + 3.

The graph is translated right 2, then reflected in x, and translated up 3.

The point (-8, 5) is transformed to (-6, -2)
(-4, -4) is transformed to (-2, 7)
(3, 2) is transformed to (5, 1)

15. Use long division to divide the polynomials: $(x^5 - 2x^4 - x^3 + 3x + 8) \div (x + 4)$

$$x^4 - 6x^3 + 23x^2 - 92x + 371$$
$$x + 4 \,\overline{)\, x^5 - 2x^4 - x^3 + 3x + 8}$$
$$\underline{-(x^5 + 4x^4)}$$
$$-6x^4 - x^3$$
$$\underline{-(-6x^4 - 24x^3)}$$
$$23x^3 + 0x^2$$
$$\underline{-(23x^3 + 92x^2)}$$
$$-92x^2 + 3x$$
$$\underline{-(-92x^2 - 368x)}$$
$$371x + 8$$
$$\underline{-(371x + 1484)}$$
$$-1476$$

$(x^5 - 2x^4 - x^3 + 3x + 8) \div (x + 4) = x^4 - 6x^3 + 23x^2 - 92x + 371$ rem: -1476

16. Use synthetic division to divide the polynomials: $(x^5 - 2x^4 - x^3 + 3x + 8) \div (x + 4)$

-4 |	1	-2	-1	0	3	8
		-4	24	-92	368	-1484
	1	-6	23	-92	371 |	-1476

$(x^5 - 2x^4 - x^3 + 3x + 8) \div (x + 4) = x^4 - 6x^3 + 23x^2 - 92x + 371$ rem: -1476

17. Use the remainder theorem to find P(-2), given $P(x) = x^5 - 2x^4 - x^3 + 3x + 8$

-2 |	1	-2	-1	0	3	8
		-2	8	-14	28	-62
	1	-4	7	-14	31 |	-54

P(-2) = -54

18. Find the real zeros of the polynomial by factoring: $2x^3 - 6x^2 - 20x$

$2x^3 - 6x^2 - 20x = 2x(x^2 - 3x - 10) = 2x(x - 5)(x + 2)$
The zeros are at x = -2, 0, 5

19. Use the zero location theorem to determine whether that $P(x) = x^5 - 2x^4 - x^3 + 3x + 8$ has a zero between -3 and -1.

-3 |	1	-2	-1	0	3	8
		-3	15	-42	126	-387
	1	-5	14	-42	129 |	-379

-1 |	1	-2	-1	0	3	8
		-1	3	-2	2	-5
	1	-3	2	-2	5 |	3

So P(-3) is negative, P(-1) is positive. There is a real zero between -3 and -1.

20. Use the rational zeros theorem to determine the possible zeros for the polynomial function $P(x) = 4x^4 - 3x^3 + 7x^2 + 4x + 6$

Possible values of p: ±1, ±2, ±3, ±6
q: ±1, ±2, ±4
The possible rational zeros are: ±1/4, ±1/2, ±3/4, ±1, ±3/2, ±2, ±3, ±6.

21. Use Descartes' Rule of Signs to determine the number of possible positive and negative zeros for $P(x) = 4x^4 - 3x^3 + 7x^2 + 4x + 6$

$P(x) = 4x^4 - 3x^3 + 7x^2 + 4x + 6$ → → 2 changes of sign, therefore 2 or 0 positive real zeros.

$P(-x) = 4x^4 + 3x^3 + 7x^2 - 4x + 6$ 2 changes of sign, therefore 2 or 0 negative real zeros.

22. The complex number $(3 + 2i)$ is a zero of the polynomial function $P(x) = x^3 + 5x^2 - 13x - 65$. Find the other zeros.

3 + 2i	1	5	-13	-65
		3 + 2i	20 + 22i	-23 + 80i
	1	8 + 2i	7 + 22i	-88 + 80i

Oops! 3 + 2i does not seem to be a zero of this polynomial. Rats!

23. Find a polynomial function of lowest degree with integer coefficients that has the zeros: $6 + 3i$, $6 - 3i$, 7.

$p(x) = (x - (6 + 3i))\ (6 - (6 - 3i))(x - 7) = (x - 6 - 3i)(x - 6 + 3i)(x - 7)$