Math 107 online practice test 2

Your exam will contain 20 questions selected from the following problem types.

1. Find the midpoint of the line segment connecting the pair of points (3,-6) and (1,–2).

   Using the midpoint formula: $\left(\frac{3+1}{2}, \frac{-6+(-2)}{2}\right) = (2,-4)$

2. Find the center and radius of the circle defined by the equation $x^2 + y^2 - 4x + 6y - 10 = 0$

   Completing the square:

   $$x^2 - 4x + y^2 + 6y = 10$$
   $$x^2 - 4x + 4 + y^2 + 6y + 9 = 10 + 4 + 9$$
   $$(x-2)^2 + (y+3)^2 = 23$$

   The circle is centered at (2,-3) and has radius $\sqrt{23}$

3. Find the equation of the circle which has center (-7,4) and radius 9.

   From the standard form of the circle: $(x + 7)^2 + (y - 4)^2 = 81$

4. Find the distance between the points (3,-6) and (1,–2).

   Using the Pythagorean theorem / distance formula:

   $$d = \sqrt{(-2-(-6))^2 + (1-3)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

5. Explain what is meant in mathematics by a *function*.

   A function is a relation in which no member of the domain is paired with more than one member of the range.

6. For the function f, where $f(x) = \frac{3x^2 - 6x}{4 - 2x^2}$, evaluate f(–5).

   Substituting x = -5

$$f(-5) = \frac{3(-5)^2 - 6(-5)}{4 - 2(-5)^2}$$
$$= \frac{3 \cdot 25 - 6(-5)}{4 - 2 \cdot 25}$$
$$= \frac{75 + 30}{4 - 50}$$
$$= -\frac{105}{46}$$

7. For the function f, where $f(x) = x^2 - 2x - 5$, simplify $f(a - h)$

Substituting $x = a - h$:

$$f(x) = (a - h)^2 - 2(a - h) - 5$$
$$= a^2 - 2ah + h^2 - 2a + 2h - 5$$

8. Find the equation of the linear function which has slope -5 and y-intercept (0,-4)

Using $y(x) = mx + b$:

$$y(x) = -5x - 4$$

9. Find the equation of the linear function which passes through the points (0,5) and (-3, 20)

To find the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 5}{-3 - 0} = \frac{15}{-3} = -5$$

So our function has the form $y(x) = -5x + b$. Given the co-ordinate (0,5), we know $b = 5$.

The equation of the function is $y(x) = -5x + 5$

10. Solve by completing the square: $x^2 + 6x - 11 = 0$

$$x^2 + 6x - 11 = 0$$
$$\Rightarrow \quad x^2 + 6x = 11$$
$$\Rightarrow \quad x^2 + 6x + 9 = 11 + 9 \qquad \textit{because } x^2 + 6x + 9 = (x + 3)^2$$
$$\Rightarrow \quad (x + 3)^2 = 20$$
$$\Rightarrow \quad (x + 3) = \sqrt{20}$$
$$\Rightarrow \quad (x + 3) = 2\sqrt{5}$$

11. State the quadratic formula.

The quadratic formula is: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

12. Solve the quadratic equation $3x^2 + 6x - 7 = 0$

The equation is not factorable: we use the quadratic formula. Use a = 3, b = 6, c = -7:

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 3 \cdot (-7)}}{2 \cdot 3} = \frac{-6 \pm \sqrt{36 + 84}}{6} = \frac{-6 \pm \sqrt{120}}{6} = \frac{-6 \pm 2\sqrt{30}}{6} = -1 \pm \frac{\sqrt{30}}{3}$$

13. Solve: $\sqrt{3-5x} - 22 = -12$

$$\begin{aligned}
&\sqrt{3-5x} - 22 = -12 \\
\Rightarrow\ & \sqrt{3-5x} = 10 \\
\Rightarrow\ & \left(\sqrt{3-5x}\right)^2 = 10^2 \\
\Rightarrow\ & 3 - 5x = 100 \\
\Rightarrow\ & -5x = 97 \\
\Rightarrow\ & x = -97/5
\end{aligned}$$

14. Solve: $\sqrt{x-2} + \sqrt{x} = 9$

$$\begin{aligned}
&\sqrt{x-2} + \sqrt{x} = 9 \\
\Rightarrow\ & \sqrt{x-2} = 9 - \sqrt{x} \\
\Rightarrow\ & \left(\sqrt{x-2}\right)^2 = \left(9 - \sqrt{x}\right)^2 \\
\Rightarrow\ & x - 2 = 81 - 18\sqrt{x} + x \\
\Rightarrow\ & -83 = -18\sqrt{x} \\
\Rightarrow\ & \sqrt{x} = \frac{83}{18} \\
\Rightarrow\ & x = \frac{83^2}{18^2}
\end{aligned}$$

15. Solve: $(2x + 3)^2 - 3(2x + 3) - 4 = 0$

Let u = 2x + 3, then:

$$u^2 - 3u - 4 = 0$$
$$\Rightarrow \quad (u - 4)(u + 1) = 0$$
$$\Rightarrow \quad u = -1, 4$$

Substituting back: 2x + 3 = -1 or 4

$$\Rightarrow \quad x = -2, 1/2$$

16. Solve: $\left(\frac{3x+2}{2x-5}\right)^2 - 3\left(\frac{3x+2}{2x-5}\right) = 10$

Let u = $\left(\frac{3x+2}{2x-5}\right)$, then:

$$u^2 - 3u - 10 = 0$$
$$\Rightarrow \quad (u - 5)(u + 2) = 0$$
$$\Rightarrow \quad u = -2, 5$$

Substituting back: $\left(\frac{3x+2}{2x-5}\right)$ = -2 or 5

$\Rightarrow$ 3x + 2 = -2(2x – 5) or 3x + 2 = 5(2x – 5)
$\Rightarrow$ 3x + 2 = -4x + 10 or 3x + 2 = 10x – 25
$\Rightarrow$ 7x = 8 or -7x = – 27
$\Rightarrow$ x = 8/7, 27/7

17. Solve: – 2(4 – 3x) – 12x > 7

$$-2(4 - 3x) - 12x > 7$$
$$\Rightarrow \quad -8 + 6x - 12x > 7$$
$$\Rightarrow \quad -8 - 6x > 7$$
$$\Rightarrow \quad -6x > 15$$
$$\Rightarrow \quad x < -5/2$$

18. Solve: (3x – 4)(2 – 5x) < 0

We know we have zeros at x = 4/3, and 2/5.
Using the method shown in the text:

Interval	Test value x	(3x – 4)(2 – 5x) < 0
(-∞,2/5)	-1	(-3 – 4)(2 – (-5)) = (-4)(2) < 0 True
(2/5, 4/3)	1	(3 – 4)(2 – 5) = (-1)(-3) < 0 False
(4/3, ∞)	2	(6 – 4)(2 – 10) = (2)(-8) < 0 True

So the solution set is (-∞,2/5) ∪ (4/3, ∞)

19. Solve: $x^2 - 5x < 14$

Rearranging the inequality: $x^2 - 5x - 14 < 0$
Working with $x^2 - 5x - 14 = 0$
$\Rightarrow \quad (x - 7)(x + 2) = 0$

So we have zeros at x = -2, and 7.
Using the method shown in the text:

Interval	Test value x	$(x - 7)(x + 2) < 0$
$(-\infty,-2)$	-3	$(-3 - 7)(-3 + 2) = (-10)(-1) < 0$ False
(-2, 7)	0	$(0 - 7)(0 + 2) = (-7)(2) < 0$ True
$(7, \infty)$	8	$(8 - 7)(8 + 2) = (1)(10) < 0$ False

So the solution set is (-2, 7)

20. Solve: $-4 < 5 - (2x - 3) < 7$

$-4 < 5 - (2x - 3) < 7$
$\Rightarrow \quad -4 < 5 - 2x + 3 < 7$
$\Rightarrow \quad -4 < 8 - 2x < 7$
$\Rightarrow \quad -12 < -2x < -1$ subtracting 8 from all sides
$\Rightarrow \quad 6 > x > ½$ dividing all sides by -2. **Don't forget to reverse the inequality signs!**

The solution set is (½, 6)

21. Sketch graphs of relations which illustrate symmetry w.r.t.
   (a) the y-axis.
   (b) the x-axis.
   (c) the origin.
   (d) the y-axis, x-axis and origin.

There are many possibilities here

22. Test the relation $x^3 + y^2 = 7$ for symmetry with respect to the x-axis and the y-axis.

To test for symmetry w.r.t the x-axis, replace y by –y:

$$x^3 + (-y)^2 = 7 \Rightarrow x^3 + y^2 = 7.$$

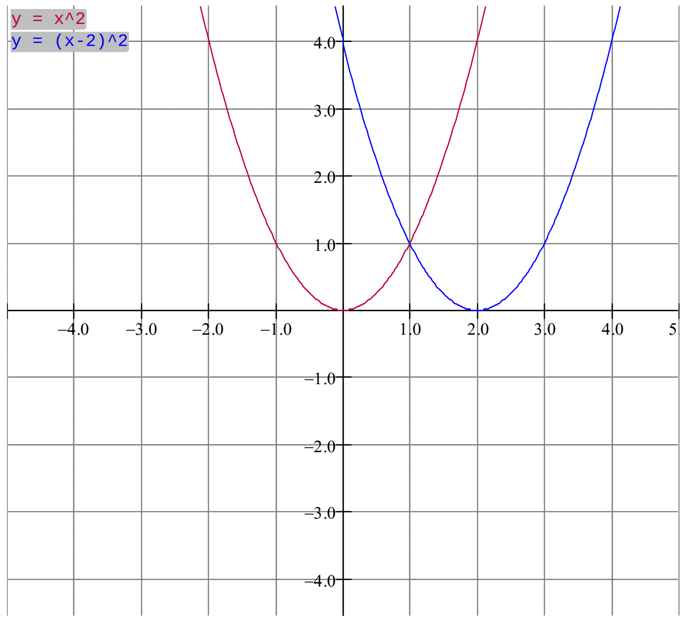
This ***is*** equivalent to $x^3 + y^2 = 7$, so we conclude that $x^3 + y^2 = 7$ ***is*** symmetric wrt x-axis.

To test for symmetry w.r.t the y-axis, replace x by –x:

$$(-x)^3 + y^2 = 7 \Rightarrow -x^3 + y^2 = 7.$$

This ***is not*** equivalent to $x^3 + y^2 = 7$, so we conclude that $x^3 + y^2 = 7$ ***is not*** symmetric wrt y-axis.

23. On a single set of axes, draw the graphs of $f(x) = x^2$ and $g(x) = (x - 2)^2$.



24. Given $f(x) = 3x – 7$ and $g(x) = 3x^2 + 2x – 4$, find
    (a) $(f – g)(-3)$
    (b) $(gf)(x)$

a) $f(-3) = 3(-3) – 7 = -9 – 7 = -16$
   $g(-3) = 3(-3)^2 – 2(-3) + 4 = 27 + 6 + 4 = 37$
   $(f – g)(-3) = f(-3) – g(-3) = -16 – 37 = -53$

b) $(gf)(x) = (3x – 7)(3x^2 – 2x + 4) = 9x^3 – 6x^2 + 12x – 21x^2 + 14x – 28$
   $= 9x^3 – 27x^2 + 26x - 28$

25. Given $f(x) =3x – 7$ and $g(x) = 3x^2 – 2x + 4$, find
    (a) $f \circ g(-2)$
    (b) $f \circ g(x)$

a) $g(-2) = 3(-2)^2 – 2(-2) + 4 = 12 + 4 + 4 = 20$
   $f \circ g(-2) = f(20) = 3(20) – 7 = 53$

b) $f \circ g(x) = f(3x^2 – 2x + 4) = 3(3x^2 – 2x + 4) – 7 = 9x^2 – 6x + 12 – 7 = 9x^2 – 6x + 5$

26. Describe the effect on the graph of $y = f(x)$ of the following transformations:
    (a) $y = f(x – 3)$
    (b) $y = -2f(x)$

a) The graph will be translated horizontally to the right 3 places
b) The graph will be stretched vertically by a factor of 2 and be reflected in the x-axis

27. If the ordered pair (-2,3) lies on the graph of y = f(x), find an ordered pair on the graph of the transformed function y = -3f(x – 2) + 7.

The graph will undergo the following transformation in order

- A horizontal translation to the right 2 places: (-2,3) $\rightarrow$ (0,3)
- A vertical stretch of factor 3 and reflection in the x-axis: (0,3) $\rightarrow$ (0,-6)
- A vertical translation up 7 places: (0,-6) $\rightarrow$ (0,1)

28. The graph of the function y = f(x) is drawn below. draw a graph of the transformed function y = -2f(x – 3) + 2.